NPTEL ONLINE CERTIFICATION COURSES

Finite Element Method

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Finite Element Method: The Course

Advanced Finite Element Analysis

- How FEM?
- What is FEM?
- Where FEM?
- Why FEM?

12 weeks

5 lectures per week

Each lecture is 25 – 30 mins

Solid Mechanics
Theory of Elasticity
Matrix Algebra
Structural Analysis
Matrix Method of Structural Analysis

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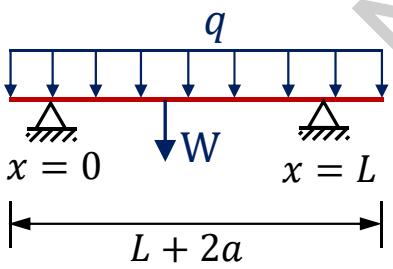
Steps in Engineering Analysis

Physical System → Idealized (simplified) System → Mathematical Model of the Idealized System → Solution of the Mathematical Model

gantry crane



$$\frac{d^2y}{dx^2} = -\frac{M(x)}{EI}$$

$$x \in (-a, L + a)$$

$$y(0) = y(L) = 0$$

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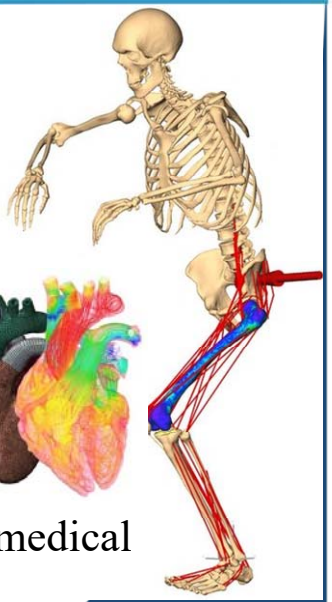
Crash analysis

Electronics

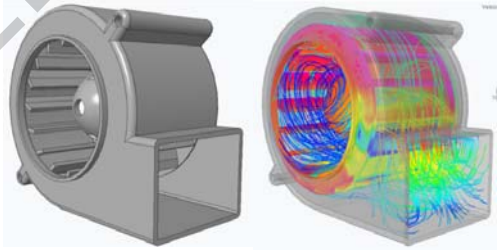
Engines and machines



Biomedical

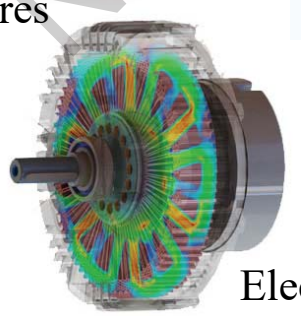
Structures under earthquake

Offshore structures



Computational fluid dynamics

Structures under moving load



Electromagnetism

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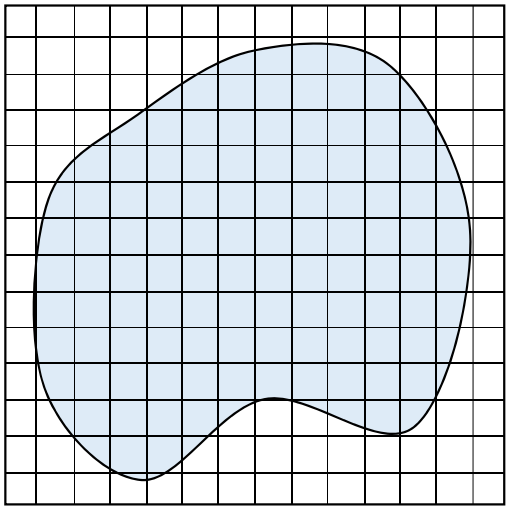
"All models are wrong, but some are useful"

George Edward Pelham Box, FRS

Approximation



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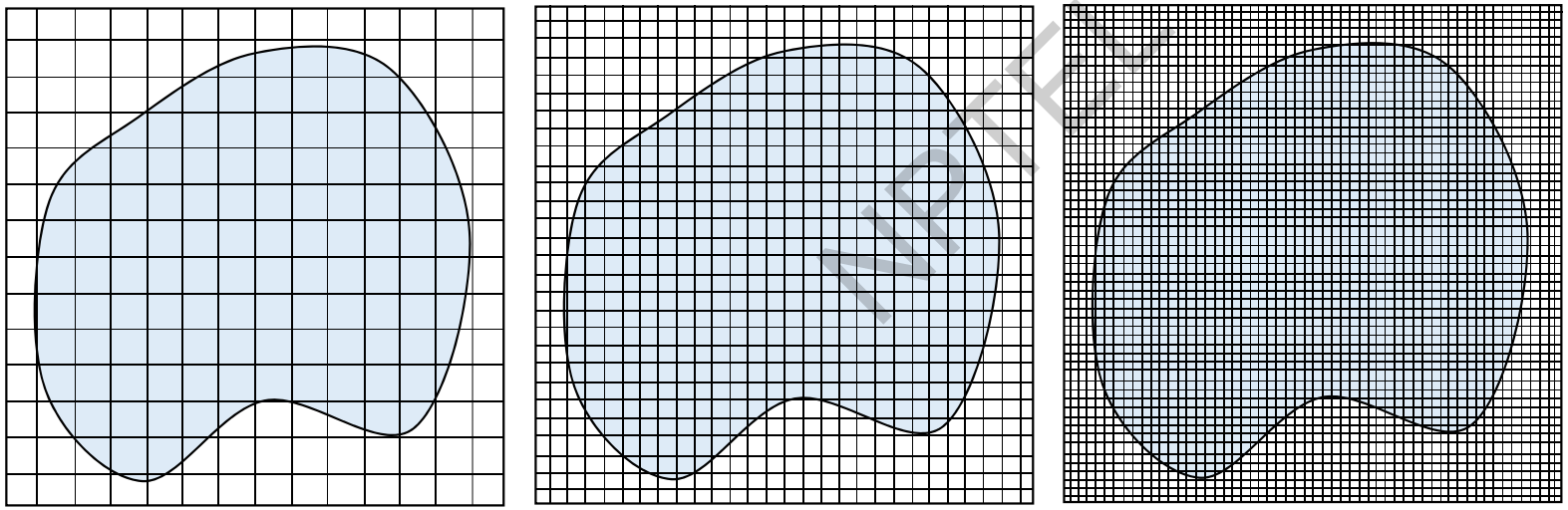
Approximation



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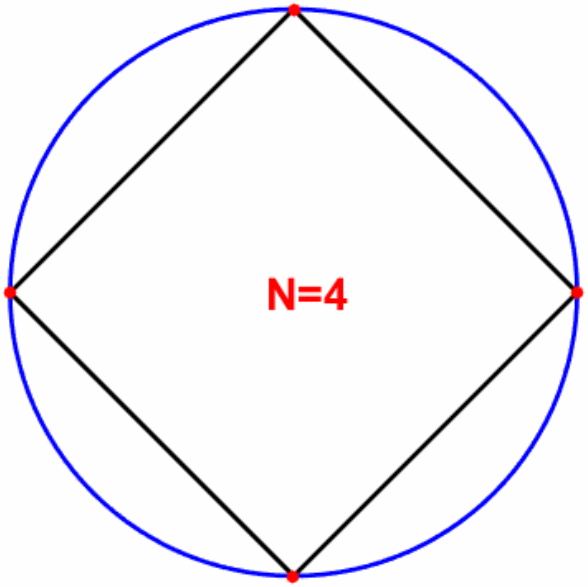
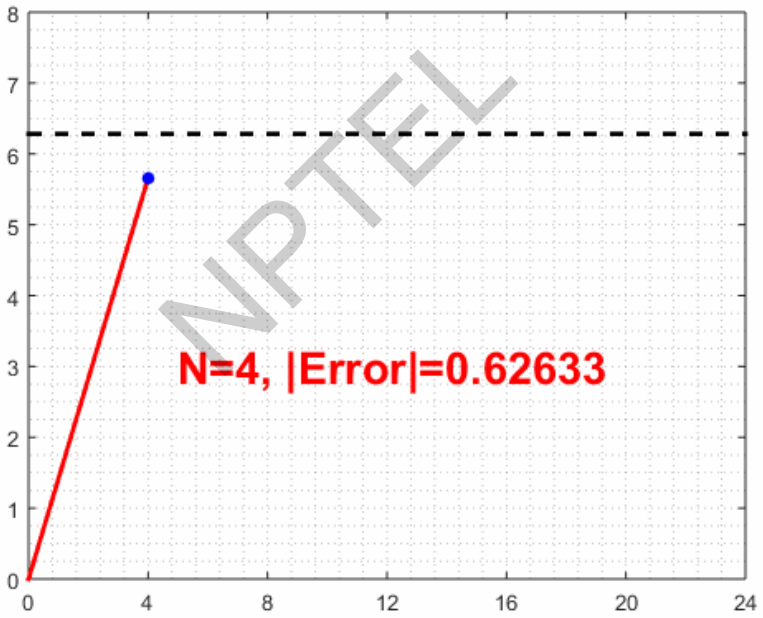
Approximation





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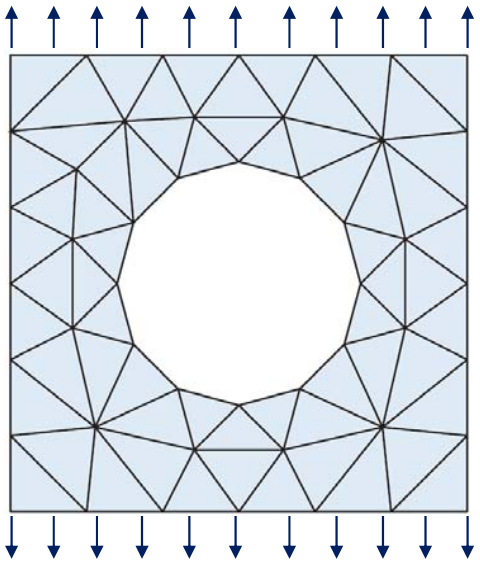
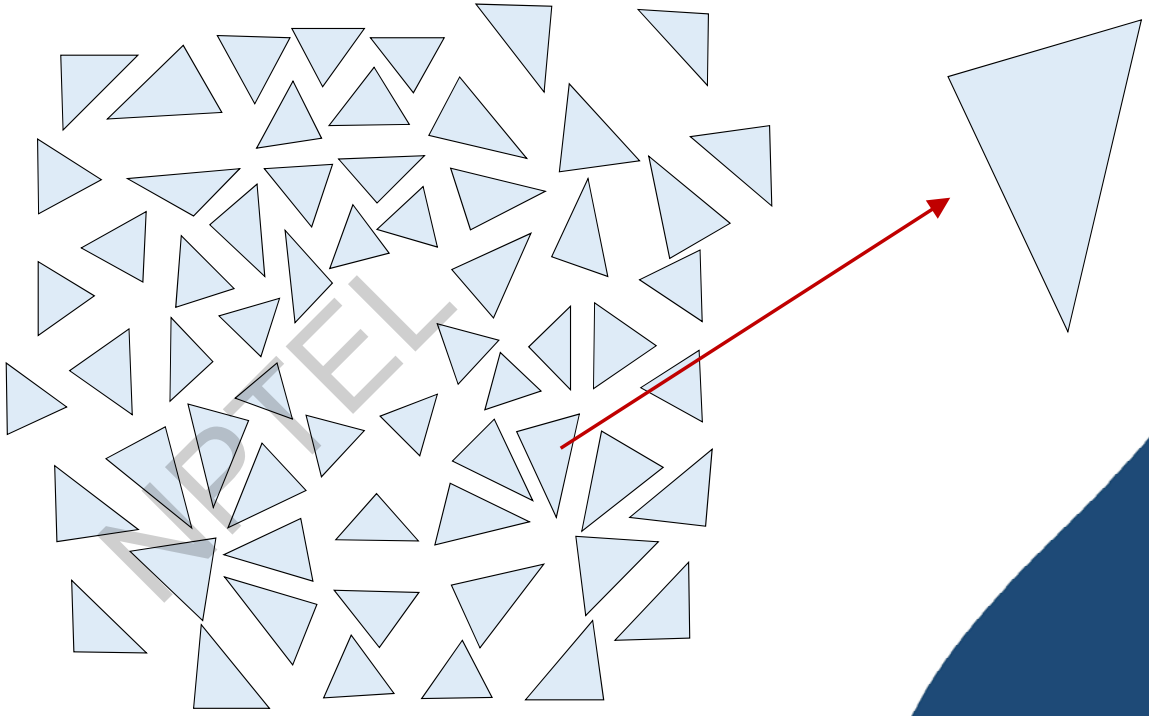
N	Order of error
100	10^{-4}
1000	10^{-5}
10000	10^{-7}
100000	10^{-9}

Finite Element Method: The Concept





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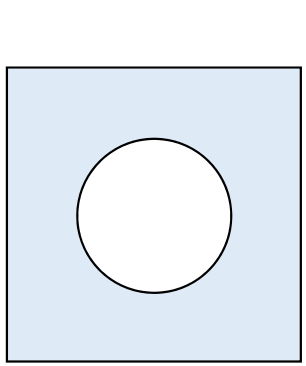
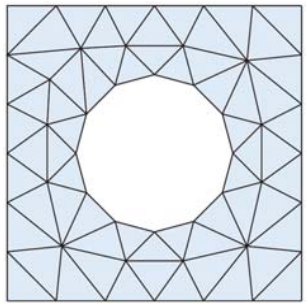
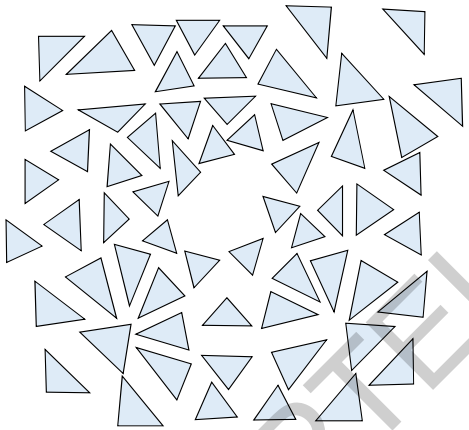
Finite Element Method: The Concept



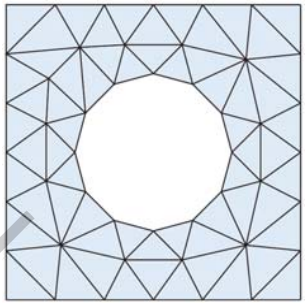
Idealized model



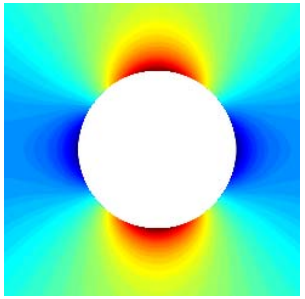
FE mesh



Element equations



Assembling of element equations to get global equation



Solution of global equation

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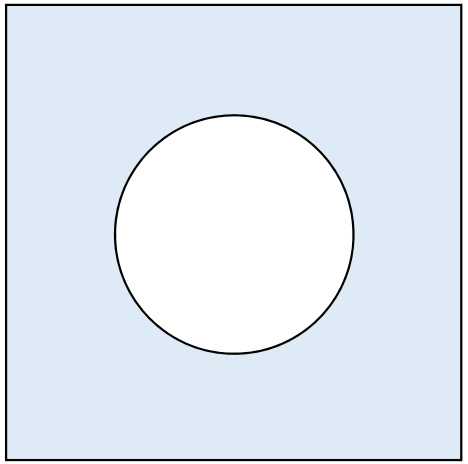
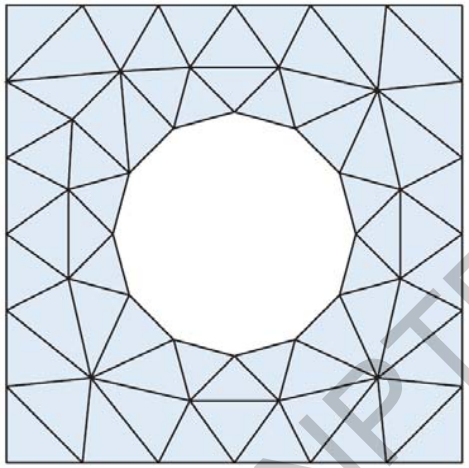
Finite Element Method: The Concept



Idealized model



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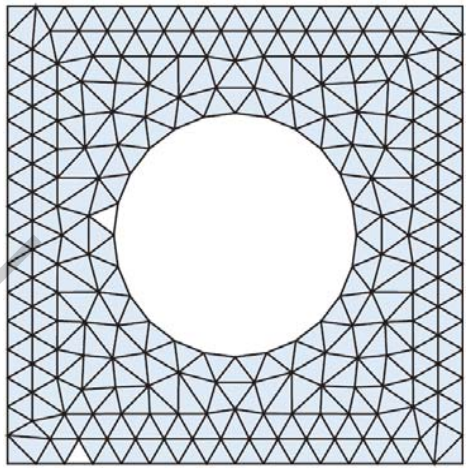

Refined FE mesh

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THANK YOU !

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Finite Element Method

Lecture 02: Introduction (Contd.)

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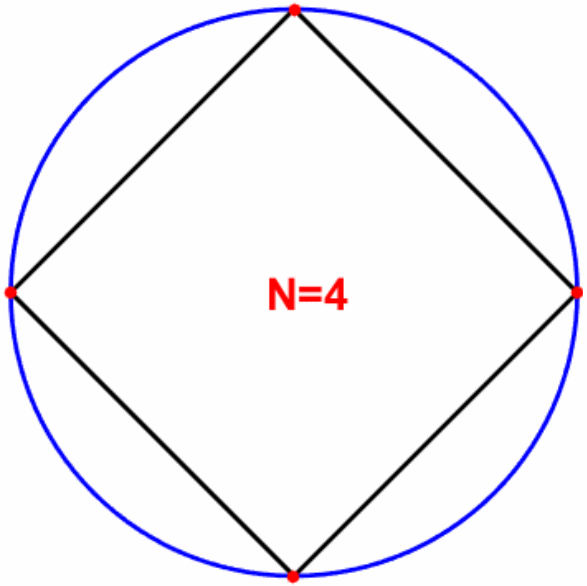

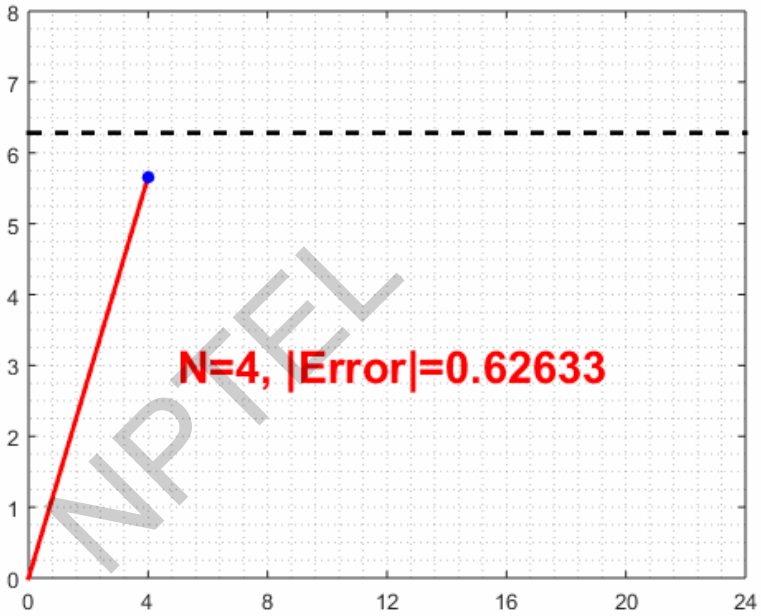

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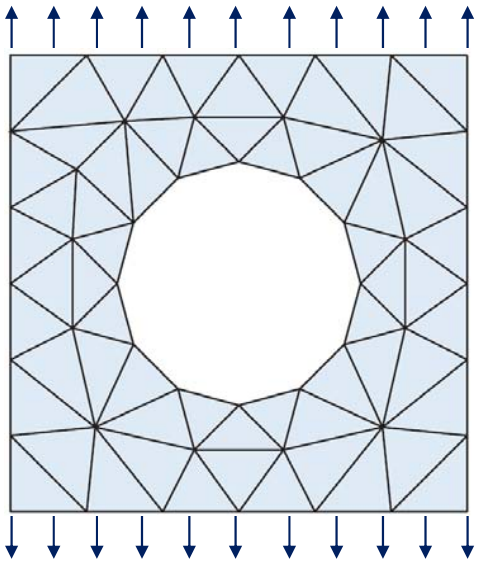
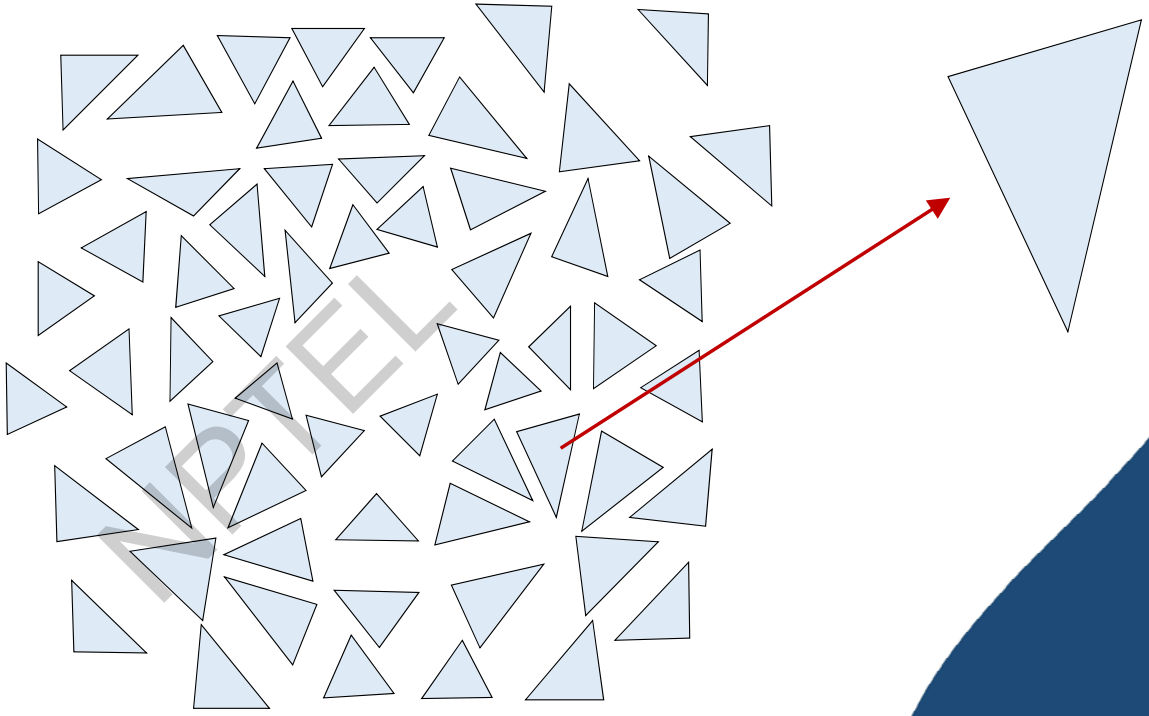
Finite Element Method: The Concept





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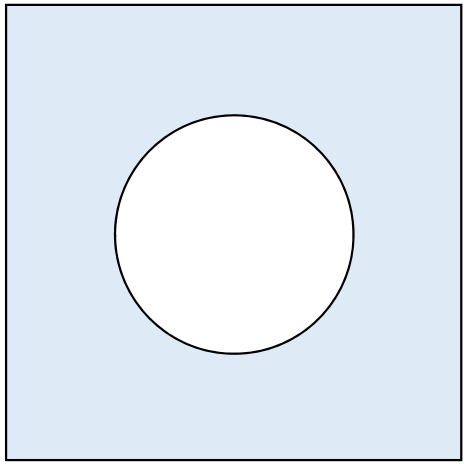
Finite Element Method: The Concept



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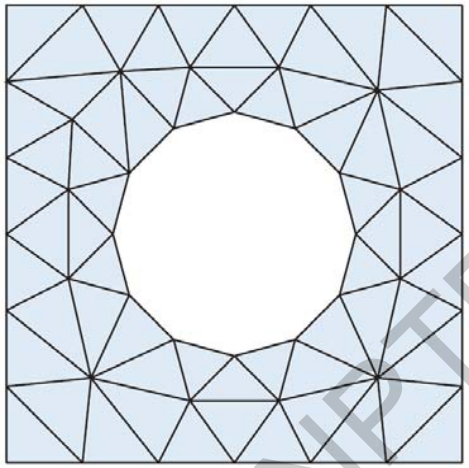
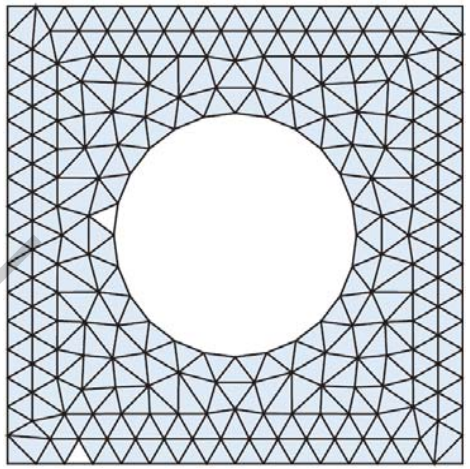


FE mesh



Refined FE mesh

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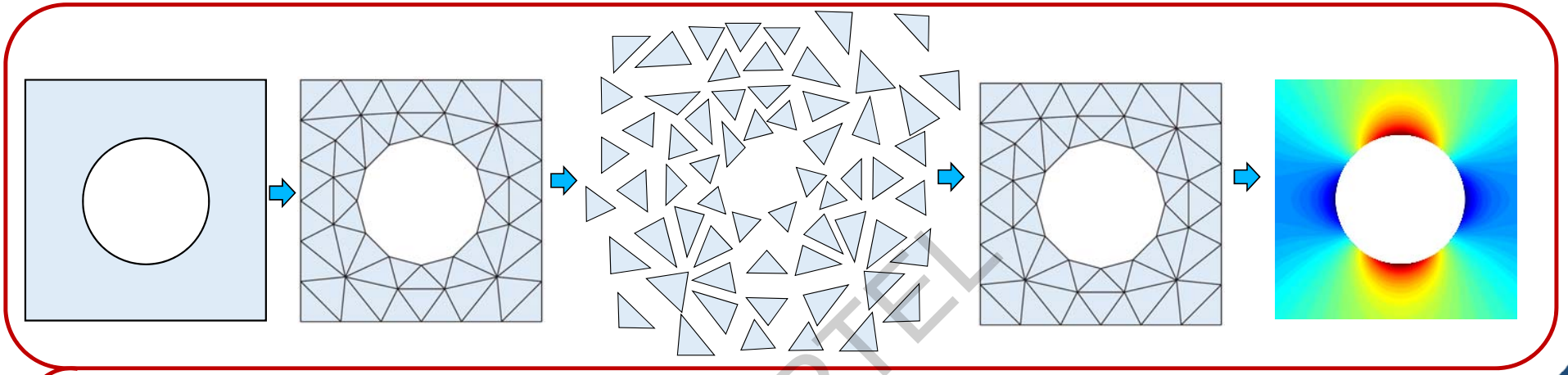
Finite Element Method: The Concept



Problem domain

Mesh generation: subdividing problem domain into finite elements

Element formulation: development of equations for elements

Assembly: Obtaining equations for the entire system from the equations of individual elements

Solution

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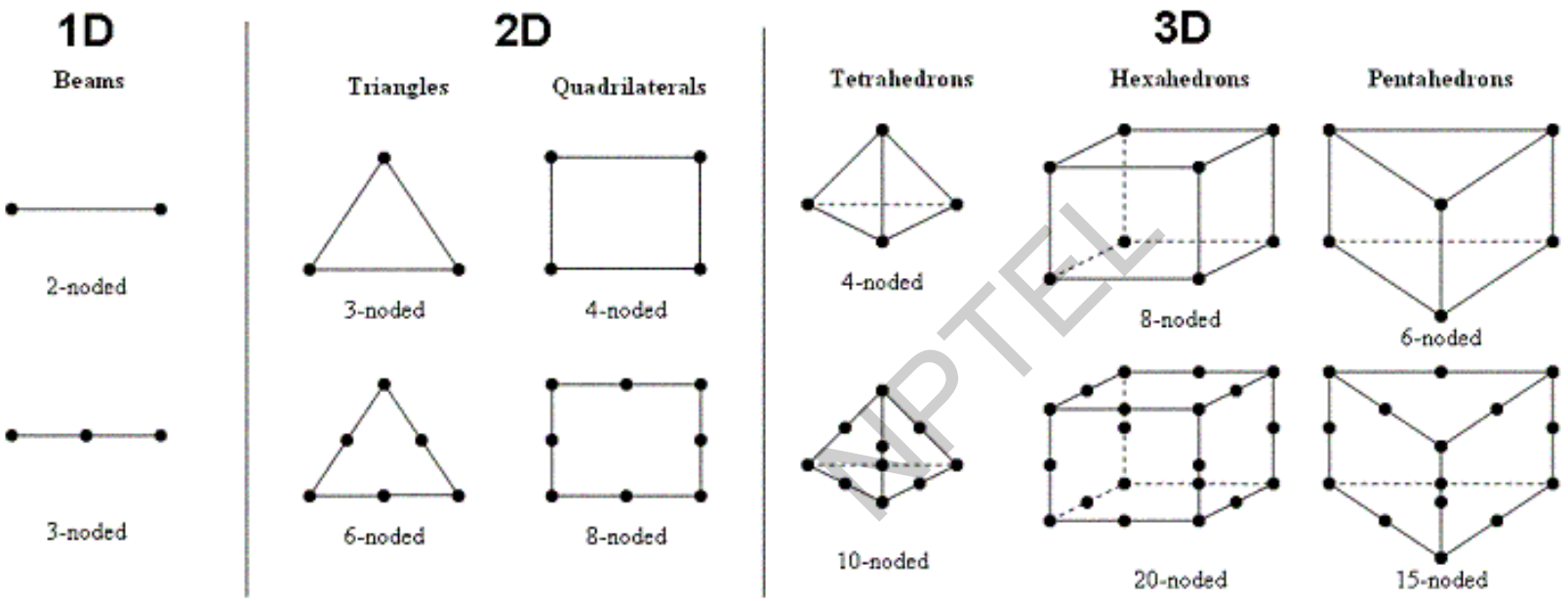
Finite Element Method: The Concept



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Finite Element Method

Lecture 03: Introduction (Contd.)

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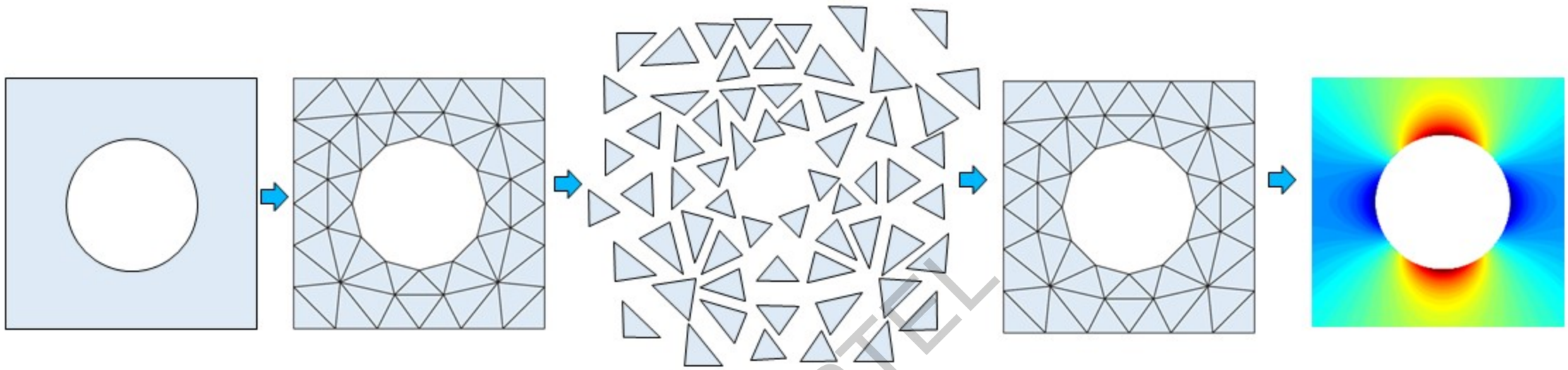
Finite Element Method: The Concept



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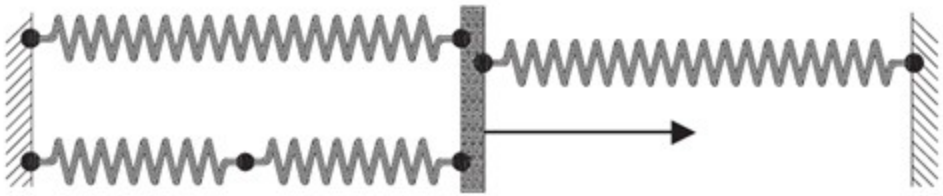

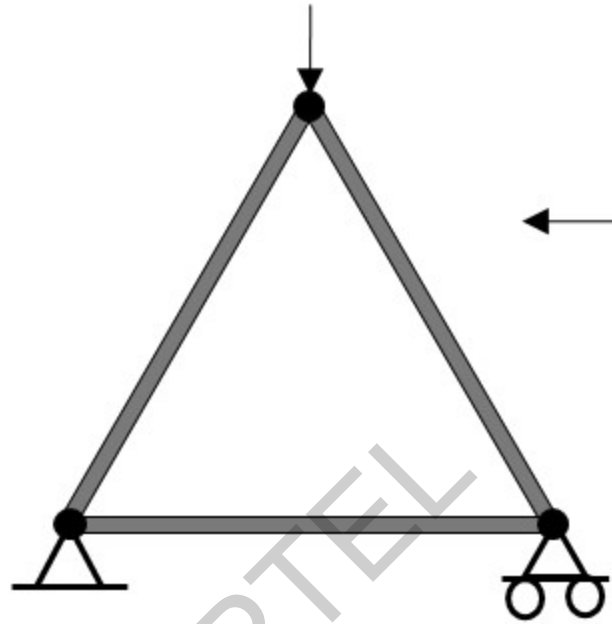

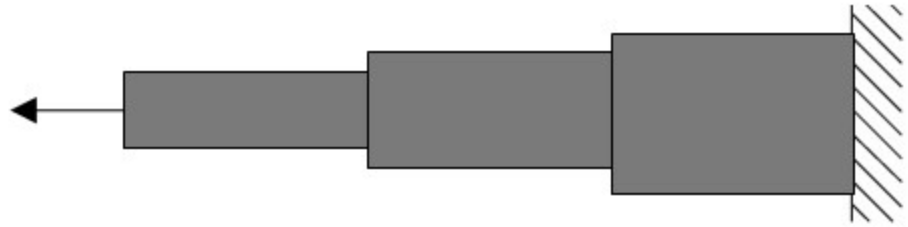

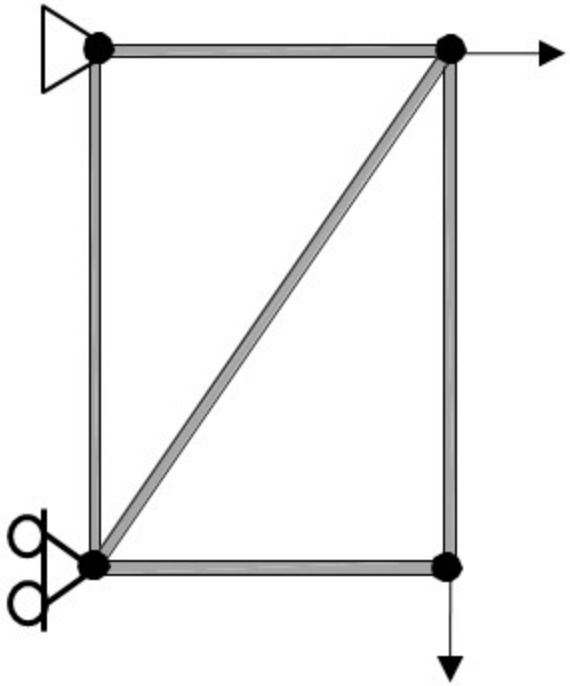

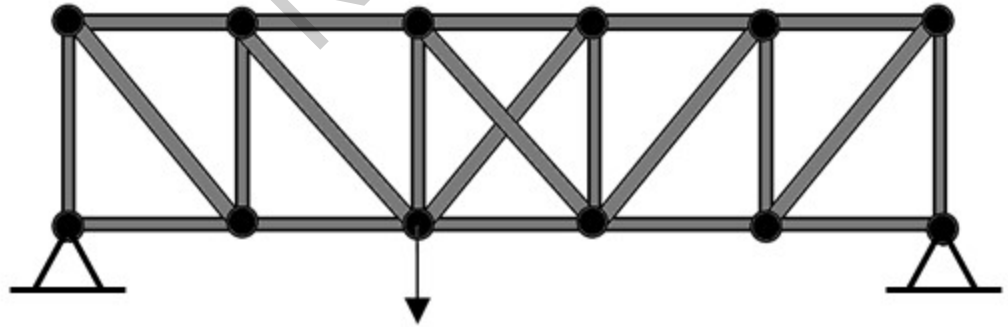

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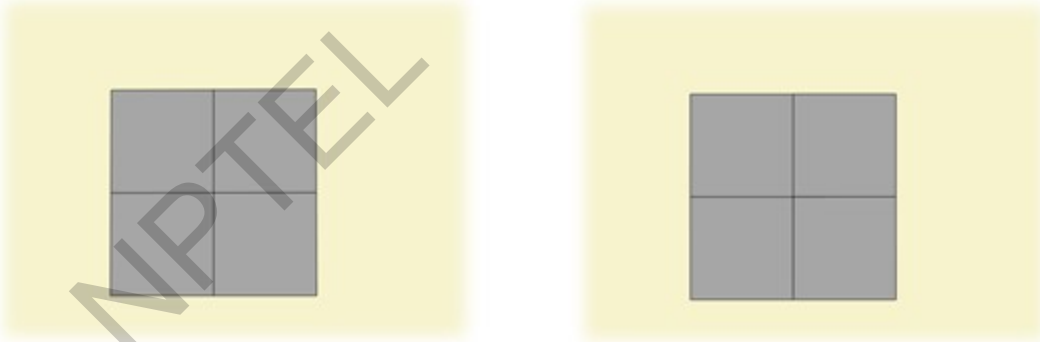
Three tenets

- Equilibrium

- Compatibility



- Constitutive Relation

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Finite Element Method

Lecture 04: Introduction (Contd.)

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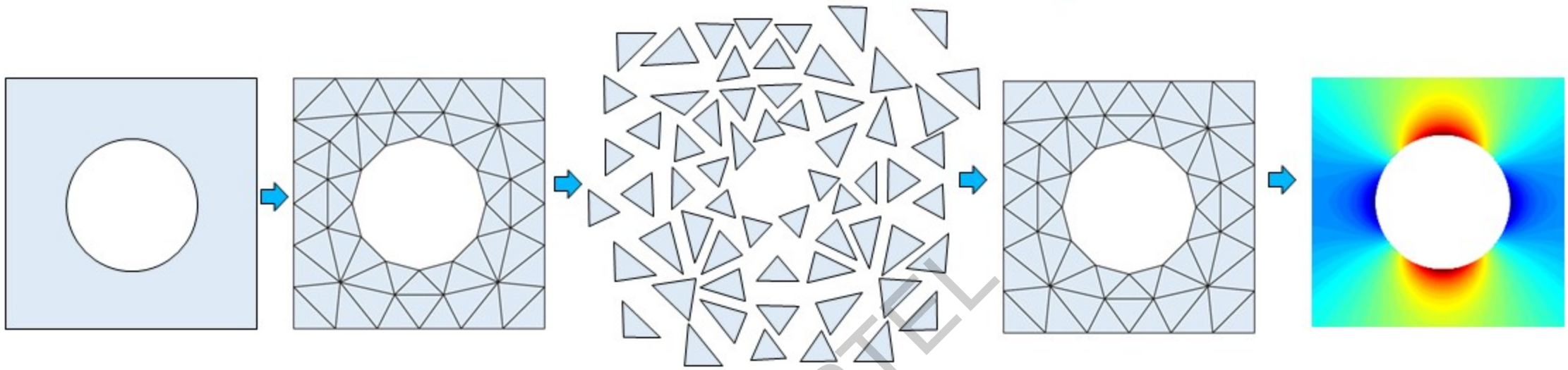
Finite Element Method: The Concept (Recall)



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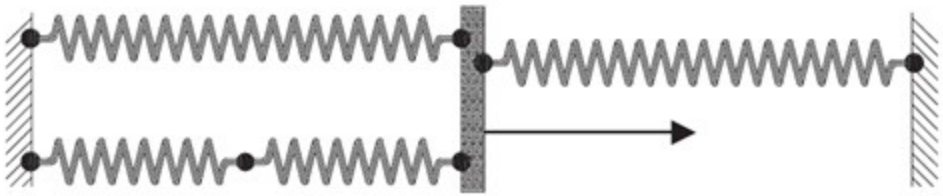

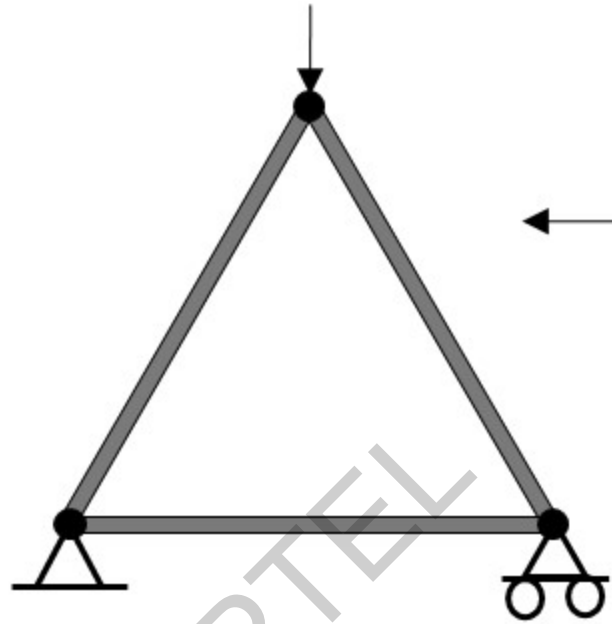

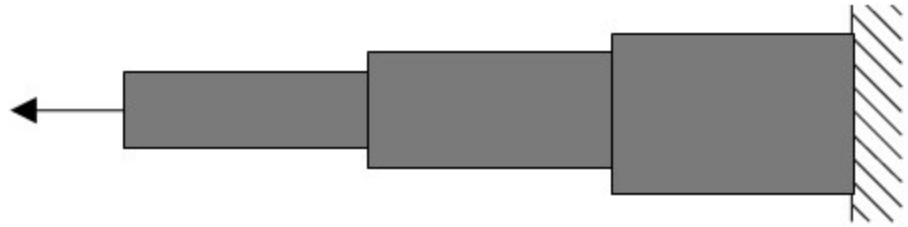

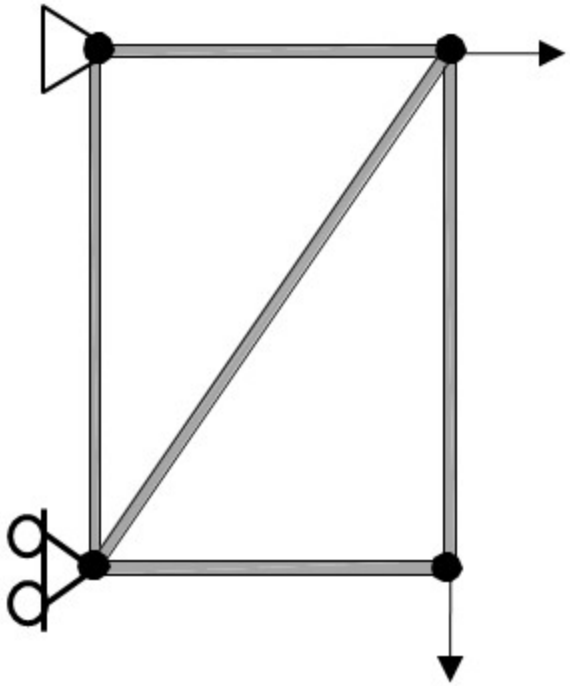

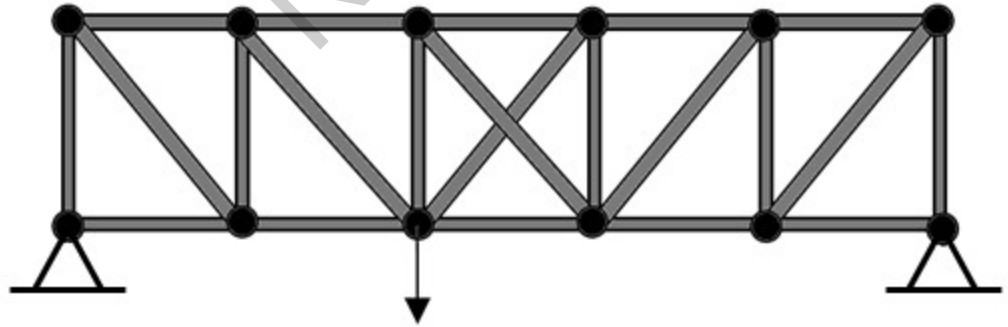

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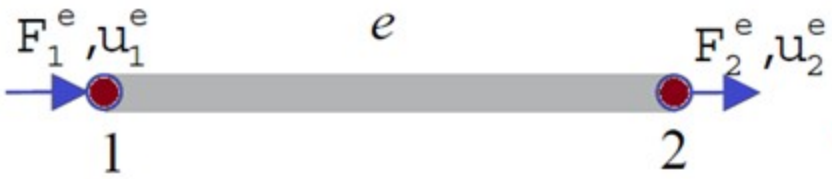
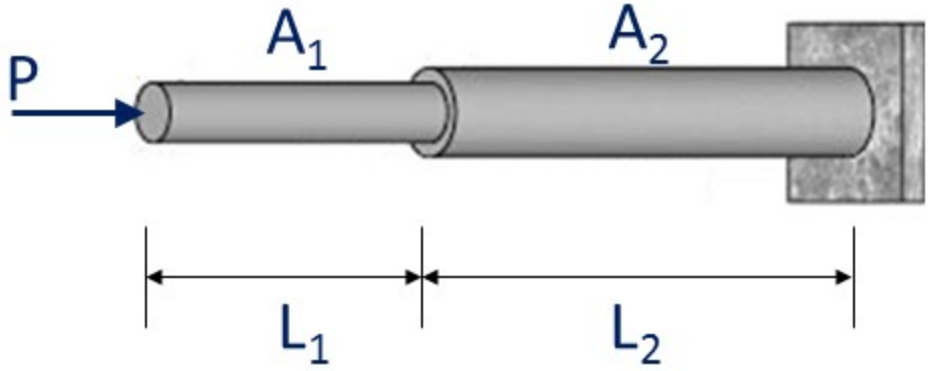
Direct Approach

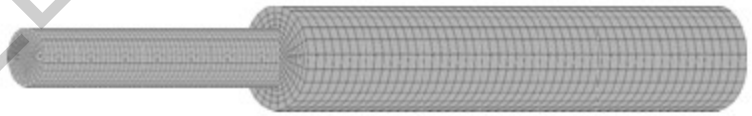


$$\underbrace{\begin{bmatrix} F_1^e \\ F_2^e \end{bmatrix}}_{\mathbf{F}^e} = \underbrace{\begin{bmatrix} k^e & -k^e \\ -k^e & k^e \end{bmatrix}}_{\mathbf{K}^e} \underbrace{\begin{bmatrix} u_1^e \\ u_2^e \end{bmatrix}}_{\mathbf{U}^e}$$

$$\mathbf{K}^e = \frac{A^e E^e}{l^e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Element Stiffness Matrix





3D idealization

1D idealization

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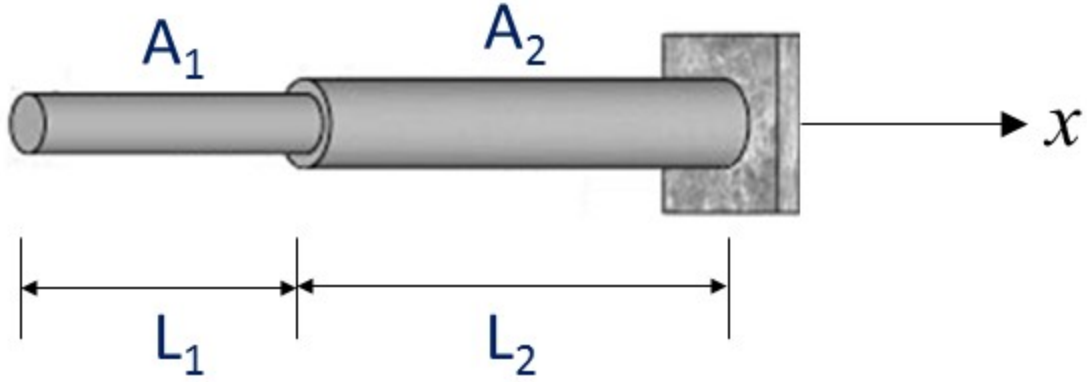

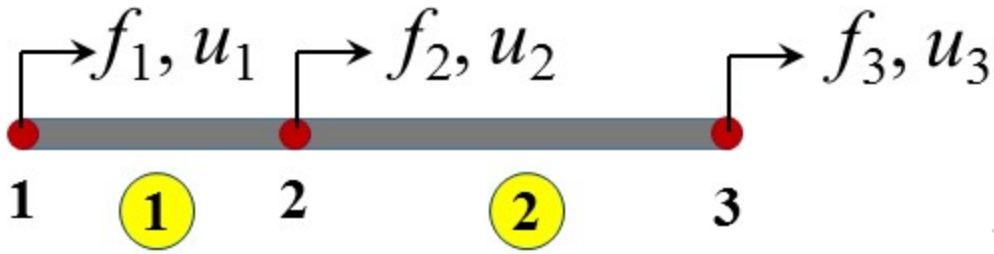

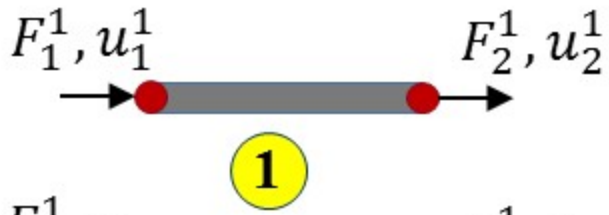

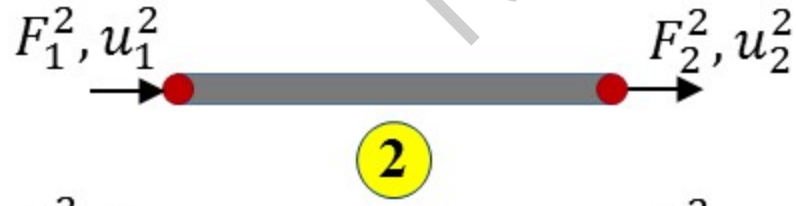

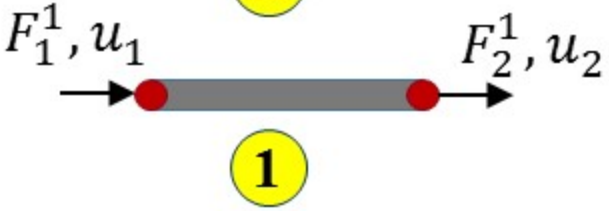

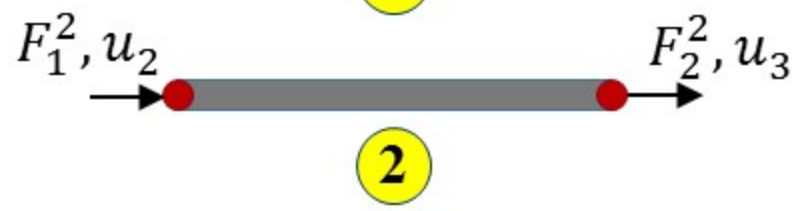

1 $\begin{Bmatrix} F_1^1 \\ F_2^1 \end{Bmatrix} = \begin{bmatrix} k^1 & -k^1 \\ -k^1 & k^1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$

2 $\begin{Bmatrix} F_1^2 \\ F_2^2 \end{Bmatrix} = \begin{bmatrix} k^2 & -k^2 \\ -k^2 & k^2 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$

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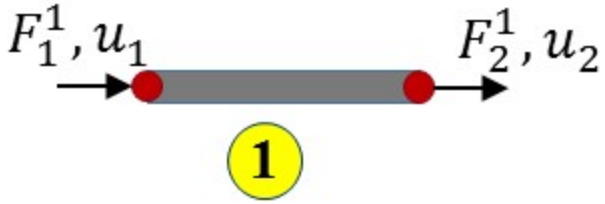
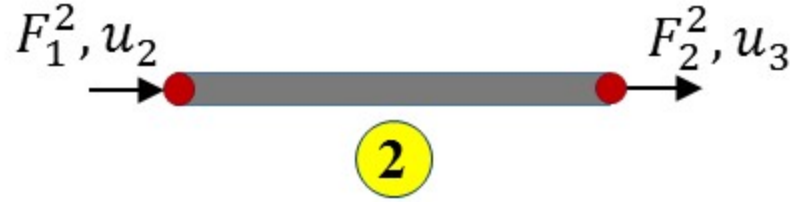


$$\begin{Bmatrix} F_1^1 \\ F_2^1 \end{Bmatrix} = \begin{bmatrix} k^1 & -k^1 \\ -k^1 & k^1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

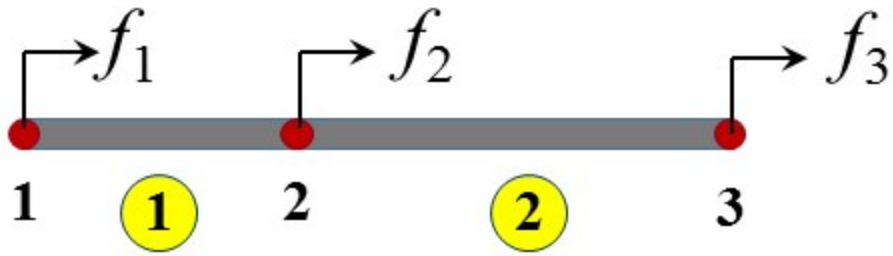
$$\begin{Bmatrix} F_1^1 \\ F_2^1 \\ 0 \end{Bmatrix} = \begin{bmatrix} k^1 & -k^1 & 0 \\ -k^1 & k^1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$



$$\begin{Bmatrix} F_1^2 \\ F_2^2 \end{Bmatrix} = \begin{bmatrix} k^2 & -k^2 \\ -k^2 & k^2 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$

$$\begin{Bmatrix} 0 \\ F_1^2 \\ F_2^2 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & k^2 & -k^2 \\ 0 & -k^2 & k^2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

NPTEL



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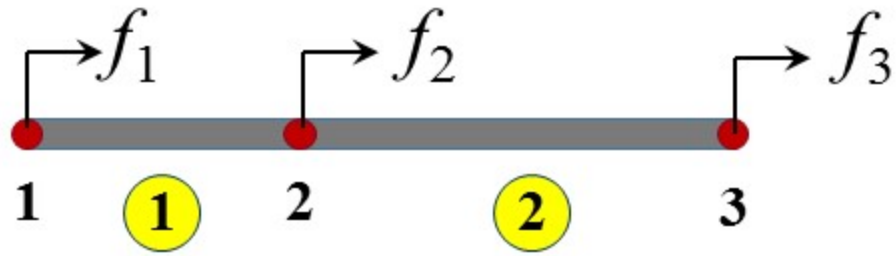
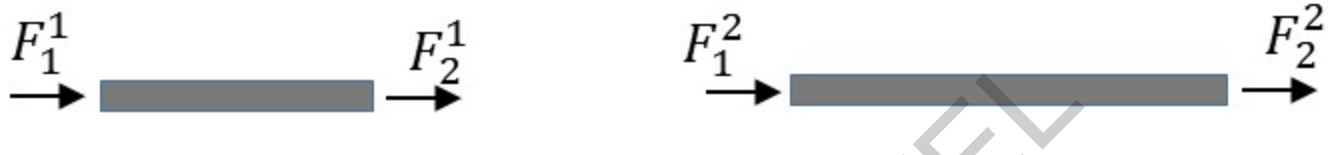
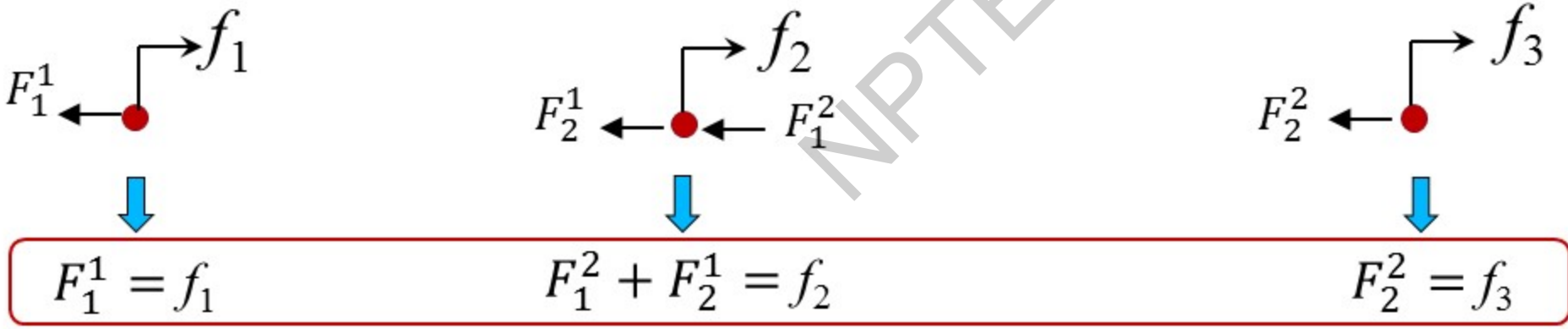


$$\begin{Bmatrix} F_1^1 \\ F_2^1 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ F_1^2 \\ F_2^2 \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \end{Bmatrix}$$





$F_1^1 = f_1$ $\quad F_1^2 + F_2^1 = f_2$ $\quad F_2^2 = f_3$

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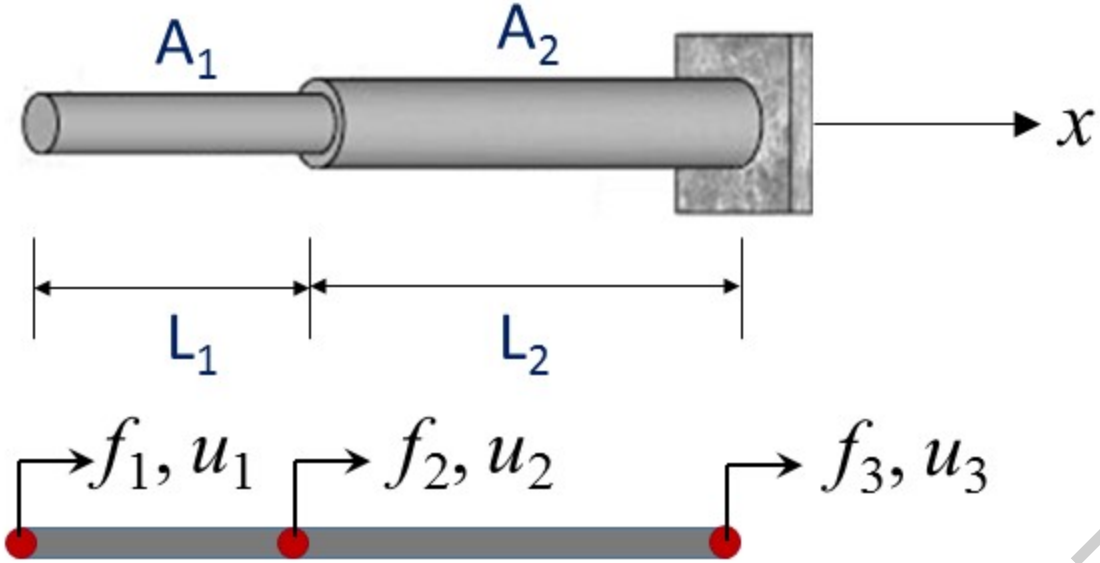

$$\begin{Bmatrix} F_1^1 \\ F_2^1 \\ 0 \end{Bmatrix} = \begin{bmatrix} k^1 & -k^1 & 0 \\ -k^1 & k^1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

$$\begin{Bmatrix} 0 \\ F_1^2 \\ F_2^2 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & k^2 & -k^2 \\ 0 & -k^2 & k^2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

$$\begin{Bmatrix} F_1^1 \\ F_2^1 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ F_1^2 \\ F_2^2 \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \end{Bmatrix} \Rightarrow \begin{bmatrix} k^1 & -k^1 & 0 \\ -k^1 & k^1 + k^2 & -k^2 \\ 0 & -k^2 & k^2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \end{Bmatrix}$$

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THANK YOU !

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